

IN MEMORY OF SHABAT ALEXEI BORISOVICH



Alexei Borisovich Shabat was born on August 8, 1937, in Moscow, in a family of scientific researchers. His father, Boris Vladimirovich Shabat, was a well-known mathematician, a professor of the chair of functional analysis in Moscow State University, an author of classical textbooks "Methods for the theory of functions of a complex variable" and "Introduction to complex analysis". His mother, Makarova Elena Alexandrovna, was a senior scientific researcher of Sternberg Astronomical Institute of the Moscow State University. During the war, all his family, together with young people from Moscow State University, was evacuated to Ashkhabad and Sverdlovsk.

In 1954-1959 A.B. Shabat studied at the Faculty of Mechanics and Mathematics of Lomonosov Moscow State University. The results he obtained being a third year student under the supervision of professor M.I. Vishik in the chair of differential equations were published in the journal "Uspekhi Matematicheskikh Nauk" (English translation is "Russian Mathematical Surveys") in 1962. His first works were devoted to boundary value problems for ordinary differential equations with a small parameter at a higher derivative, to the theory of elliptic equations and to solving a series of problems in the classical hydrodynamics. In 1963, he defended a PhD thesis "On gluing potential and vortex motion of a fluid" in the Institute of Mathematics of the Siberian Branch of Academy of Sciences of USSR in Novosibirsk under the supervision of academician M.A. Lavrentiev. In 1974, he successfully defended a Habilitation thesis "Operators of transformations and nonlinear equations" in the Faculty of Mechanics and Mathematics of Moscow State University.

After graduating from Moscow State University, he moved to Akademgorodok Novosibirsk, where he worked in the Institute of Hydrodynamics of the Siberian Branch of

Academy of Sciences of USSR in a theoretical department headed by L.V. Ovsiannikov and in the chair of differential equations headed by S.L. Sobolev in Novosibirsk State University (1959–1973). In end of 60s, as a result of collaboration with V.E. Zakharov, in the Academic Campus there arose a scientific direction focused on applying the method of scattering inverse problem in the modern mathematical physics. In Ufa, where A.B. Shabat moved in the end of 1973, he spent 15 years of most fruitful scientific life. First, we worked in Bashkir State University in the chair of differential equations headed by professor M.D. Ramazanov (1974–1990), then in the Department of Physics and Mathematics headed by professor V.I. Khvostenko of Bashkir Branch of the Academy of Sciences of Russia. In Ufa, A.B. Shabat succeeded to create a scientific group aimed on developing the ideas by Sophus Lie in the theory of solitons. In the beginning of 80s, the obtained results ensured an international recognition of this group of scientists and played their role in a boost forming of the mathematical aspects of the theory of solitons as well as of the notion of integrability. In 1990, A.B. Shabat left Ufa and continued his work in the Landau Institute of Theoretical Physics of RAS in Chernogolovka. During all his life he kept close scientific collaboration with his Ufa scientific school. From 2007 till 2018 he worked in Karachay-Cherkess State University named after U. D. Aliyev, where he headed scientific seminars and supervised PhD students. From 2018, he was a scientific researcher of Caucasus Mathematical Center at Adyghe State University.

A.B. Shabat gained a world fame and a recognition not only of mathematicians but also of theoretical physicists thanks to his fundamental results in the modern theory of integrable systems related with the developing of the method of scattering inverse problem, a gem of the mathematical physics in XX century. He made a fundamental contribution to the developing of the theory of solitons, a new method in the modern mathematical physics.

Starting from 1969, the scientific interests of A.B. Shabat are focused on various issues related with a new approach to the theory of nonlinear waves discovered by a group of American theoretical physicists in 1967.

In 1970–1979, jointly with V.E. Zakharov, they created and developed a general scheme of integrating nonlinear differential equations by the method of inverse scattering problem known now as the "dressing method" or Zakharov-Shabat method. Exactly after a famous work V.F. Zakharov, A.B. Shabat, Exact theory of two-dimensional self-focusing and one-dimensional self-modulation of wave in nonlinear media, *Zhurn. Exper. Teor. Fiz.* 61:1, 118–134 (1971) [*JETP.* 34:1, 62–69], the "method of inverse scattering problem" became a method. In those years, A.B. Shabat published also a series of pioneering works developing the method of inverse scattering problem and used first the Riemann-Hilbert problem for solving the inverse scattering problem.

In end of 70s, he began solving problems on classification of integrable equations. A working group was created in Ufa for working on this project. Apart of the pupils of A.B. Shabat (A.V. Zhiber, V.V. Sokolov, I.T. Habibullin, S.I. Svinolupov, R.I. Yamilov, V.E. Adler, I.Yu. Cherdantsev, B.A. Magadeev), in different years, N.Kh. Ibragimov, A.N. Leznov, A.V. Mikhailov, and F.Kh. Mukminov participated in the work of that group. As a result of their work, simple and effective integrability criteria were formulated being necessary conditions for the existence of higher symmetries and conservation laws.

In 1974, in Ufa, A.B. Shabat organized a well-known, first in Russia conference on theory of solitons and the method of the inverse scattering method, which gathered both a pleiad of outstanding scientists and a young generation.

In 80s, on the base of a Shabat theorem on existence of a generalized Lax pair for evolution equations possessing higher symmetries, a symmetry approach to the integrability problem was developed. In collaboration with pupils, they developed effective integrability criterions and provided a complete description and a classification of integrable nonlinear equations generalizing an anisotropic Landau-Lifschitz model. Integrable systems of

nonlinear Schrödinger equation type were completely described and classified. We also mention a cycle of works by A.B. Shabat made jointly with his pupils R.I. Yamilov and V.A. Adler in 1987–2000, in which they completed a classification of Lagrange nonlinear chains with interacting closest neighbours.

Works made by A.B. Shabat in 90s, were devoted mostly to developing a theory of discrete symmetries. He developed a rather general scheme for discretization of spectral problems and studied lattice equations for main spectral problems. As an application of that theory, A.B. Shabat pointed out new exactly solvable problems in one-dimensional quantum mechanics with "arithmetical" spectra and established a series of interesting facts for equations of Painlevé type.

Recently, scientific interests of A.B. Shabat were concentrated on classical problem on commuting differential operators in a multi-dimensional case.

In 1996–1999, A.B. Shabat, jointly with V.E. Zakharov, obtained a grant as a head of the direction "Mathematical theory of exactly integrable nonlinear models" of a leading scientific school "Theory of nonlinear waves".

A.B. Shabat was a coordinator of Einstein consortium, which organized a series of joint conferences NEEDS in Italy and Russia. In 2001, he was invited to Newton Mathematical Institute in Cambridge as a Rothschild Visiting Professor. In different years, A.B. Shabat worked in the universities in Rome, Madrid, Minnesota, Loughborough, Leeds, Montpellier and others.

There are more than 10 candidates of sciences (equivalent to PhD) and 6 doctors of sciences (equivalent to Habilitation) among his pupils. Ufa mathematicians are deeply grateful to Alexei Borisovich Shabat and to Arlen Mikhailovich Il'in, who worked in Ufa, for creating in Ufa groups of an international level in the scientific directions "Integrability theory" and "Asymptotics of singularly perturbed problems". Thanks to their efforts and mathematical discoveries, mathematical schools appeared in Ufa, which together with a known school of A.F. Leontiev on the theory of functions of complex variables led to a foundation of Institute of Mathematics with Computer Center of Ufa Scientific Center of RAS. The institute still works successfully nowadays.

Alexei Borisovich Shabat passed away on March 24, 2020 and was buried in Maykop. A blessed memory of Alexei Borisovich will stay forever in the hearts of the people knew him.

Colleagues and friends.