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CENTRAL LIMIT THEOREM FOR RANDOM EXPONENTS ON HILBERT SPACE IN WEAK OPERATOR TOPOLOGY

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Abstract. We consider random linear continuous operators $\Omega \to \mathcal{L}(\mathcal{H}, \mathcal{H})$ on a Hilbert space $\mathcal{H}$. For example, such random operators may be random quantum channels. The Central Limit Theorem is known for the sums of i.i.d. random operators. Instead of the sum, there may be considered the composition of random exponents $e^{A_i/n}$. We obtain the Central Limit Theorem in the Weak Operator Topology for centralized and normalized random exponents of i.i.d. linear continuous operators on a Hilbert space.

Keywords: random operators, central limit theorem, weak operator topology.

Mathematics Subject Classification: 28B05; 47B02; 60F05.

1. Introduction

The theory of compositions of random linear operators is constructed and developed in many publications [11], [21], [34]. A random quantum dynamics can be considered either as a composition of random linear operators in a Hilbert space [18], [23], or as a composition of random quantum channels [1], [7], [14], [19], [28]; in [1], [3], [13], [14], [17], [19], [22], [24], [29], [30] one can see how random operator–valued processes arise.

The Law of Large Numbers (LLN) and the Central Limit Theorem (CLT) for random variables acting in a Banach space is well studied [15], [20]. Distributions of products of random linear operators were studied in [27], [28], [33], [34]. Different versions of limit theorems of LLN– and CLT–type for compositions of i.i.d. random operators were obtained in [4], [8], [9], [10], [14], [25], [26], [32], [35], [36].

Here are some versions and approaches for the CLT–type problems.

- In [2] the convolutions of quantum states were considered. It was shown that the convergence rate of these convolutions to a Gaussian state in the Schatten 1-norm is $O(1/\sqrt{n})$.
- In [26] it was shown that the weak convergence of measures is equivalent to the pointwise convergence of convolution operators. Different topologies of test functions instead of continuous bounded functions were considered. In those topologies the CLT–type results were obtained for the random operator–valued processes.
- In [4] there were considered the random matrices of the kind $I + \frac{A_i}{\sqrt{n}} + \frac{B_i}{n} + O(n^{-3/2})$. It was shown there that their products converge uniformly (in distribution) to the log–normal random distribution given by some stochastic equation.
- In [35, 36] the composition $e^{A(\xi_1)/n} \dots e^{A(\xi_{\lfloor n^2 t \rfloor})/n}$ was considered. The weak convergence of these processes was proved.

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In [10], the compositions of random semigroups of the form $e^{A_i t/n}$ for operators on Banach spaces were studied. The Strong Law of Large Numbers was obtained for them in the spirit of Chernoff Theorem [5]; it was established in the Weak Operator Topology for all Banach spaces, and in the Strong Operator Topology for uniformly smooth Banach spaces. Following [26], it is natural to centralize the random compositions in [10], multiply them by $\sqrt{n}$, and obtain the CLT for them as is done in classical Probability. In this paper, we obtain such CLT for operators on a Hilbert space. Namely, we establish that the sequence

$$\sqrt{n}\left(e^{A_1/n}\dots e^{A_n/n}-e^{\mathbb{E}\,A}\right)$$

of operators converges in distribution in the Weak Operator Topology to a random Gaussian operator.

The paper is organized as follows. In Section 2 we formulate the main result, Theorem 2.1. This theorem is proved in Section 3.

2. Background and statements

Let $(\Omega,\mathcal{F},\mathrm{P})$ be a probability space with a complete σ-additive measure P.

For a Hilbert space $\mathcal{H}$ by $\mathcal{L}(\mathcal{H})$ we denote the space of bounded linear operators on $\mathcal{H}$. The map $A\colon\Omega\to\mathcal{L}(\mathcal{H})$ is called a *random operator* if it is *Bochner measurable*, i.e. if there exists a sequence of finitely–valued maps $\Omega\to\mathcal{L}(\mathcal{H})$ converging to A uniformly almost surely. The *expected value* $\mathbb{E}\,A$ of a random operator is its *Bochner integral*. It is well–known [6], [37] that $\mathbb{E}\,A$ exists if and only if the random variable $\|A\|$ is Lebesgue integrable.

The independence of random operators is defined similarly to the independence of random variables. It was proved in [10] that the composition of random operators is a random operator, and that the expected value of the composition of independent random operators is the composition of their expected values.

For a linear bounded positive symmetric operator Σ on $\mathcal{H}^{\otimes 2}$ we denote by $N(0,\Sigma)$ the distribution of a zero–mean random Gaussian operator G with the covariance Σ; that is, for all $x,y\in\mathcal{H}$ the random variable $\langle y,Gx\rangle$ is a zero–mean Gaussian variable with the variance equalling to $\langle y^{\otimes 2},\Sigma x^{\otimes 2}\rangle$.

Theorem 2.1. *Let $\mathcal{H}$ be a Hilbert space over $\mathbb{R}$. Let $A\colon\Omega\to\mathcal{L}(\mathcal{H})$ be such that $\|A\|$ is bounded. Let A, A_1, $A_2,\dots$ be a sequence of i.i.d. random operators. Then the sequence*

$$\sqrt{n}\left(e^{A_1/n}\dots e^{A_n/n}-e^{\mathbb{E}\,A}\right)$$

of random operators converges in distribution in WOT to $N(0,\Sigma)$ as $n\to\infty$, where

$$\Sigma:=\int_{s=0}^{1}\left(e^{\mathbb{E}\,As}\right)^{\otimes 2}\mathbb{E}(A-\mathbb{E}\,A)^{\otimes 2}\left(e^{\mathbb{E}\,A(1-s)}\right)^{\otimes 2}ds$$

is the Bochner integral. In other words, for all $x,y\in\mathcal{H}$ we have

$$\left\langle y,\sqrt{n}\left(e^{A_1/n}\dots e^{A_n/n}-e^{\mathbb{E}\,A}\right)x\right\rangle\xrightarrow[n\to\infty]{d}N\left(0,\left\langle y^{\otimes 2},\Sigma x^{\otimes 2}\right\rangle\right).$$

Note that it was shown in [10, Thm. 2.5] that $\sup\limits_{t\in[0,T]}\left|\left\langle y,\left(e^{A_1t/n}\dots e^{A_nt/n}-e^{\mathbb{E}\,At}\right)x\right\rangle\right|$ converges almost surely to 0. Therefore, it would be interesting to obtain the convergence in distribution as in Theorem 2.1, but for the sup for semigroups instead of just exponents, or as in [4].

3. Proof of the main result

Lemma 3.1 (cf. [12, Thm. 3]). *Let $\mathcal{H}$ be a Hilbert space. Let $A\colon \Omega \to \mathcal{L}(\mathcal{H})$ such that $\|A\|$ is bounded. Then for any $n \in \mathbb{N}$ and $1 \leqslant k \leqslant n$*

$$\left\| \left(\mathbb{E}\, e^{A/n}\right)^k - e^{\mathbb{E}\, Ak/n} \right\| = O(1/n) \quad \text{as } n \to \infty,$$

with the constant in $O(1/n)$ independent of $k = k(n)$.

Proof. By the Taylor formula for Banach spaces, both $\mathbb{E}\, e^{A/n}$ and $e^{\mathbb{E}\, A/n}$ can be expressed as $I + \frac{\mathbb{E}\, A}{n} + O\left(1/n^2\right)$. Then

$$\left\|\mathbb{E}\, e^{A/n} - e^{\mathbb{E}\, A/n}\right\| = O\left(1/n^2\right).$$

Now the result follows from the formula

$$X^k - Y^k = \sum_{j=0}^{k-1} X^{k-j-1}(X-Y)Y^j$$

with $X := \mathbb{E}\, e^{A/n}$ and $Y := e^{\mathbb{E}\, A/n}$. The proof is complete. □

Proof of Theorem 2.1. We fix an arbitrary $x \in \mathcal{H}$ and denote

$$\xi_n := \sqrt{n}(e^{A_1/n} \dots e^{A_n/n} - e^{\mathbb{E}\, A}) \quad \text{and} \quad S_n := \frac{1}{\sqrt{n}} \sum_{k=1}^{n} e^{\mathbb{E}\, A\frac{k-1}{n}}(A_k - \mathbb{E}\, A)e^{\mathbb{E}\, A\frac{n-k}{n}}.$$

We need to prove that $\|\xi_n x - S_n x\| \xrightarrow[n\to\infty]{\mathrm{P}} 0$, and that S_n is a triangular array of martingales with respect to $\mathcal{F}_{n,k} = \mathcal{F}_k = \sigma(A_1, \dots, A_k)$ for which the following conditions hold:

$$\sum_{k=1}^{n} \mathbb{E}^{\mathcal{F}_{k-1}}\, d_{n,k} \otimes d_{n,k} \xrightarrow[n\to\infty]{\mathrm{P}} \Sigma, \tag{3.1}$$

$$\forall \varepsilon > 0 \quad \sum_{k=1}^{n} \mathbb{E}^{\mathcal{F}_{k-1}} \left\|d_{n,k}\right\|^2 \mathcal{I}\{\|d_{n,k}\| > \varepsilon\} \xrightarrow[n\to\infty]{\mathrm{P}} 0, \tag{3.2}$$

where $d_{n,k}$ are the summands in S_n. Then the result will follow by CLT [16, Cor. 3.1] for real–valued martingales.

Let us show that S_n is a triangular array of martingale differences. Indeed, $\mathbb{E}^{\mathcal{F}_{k-1}}\, d_{n,k} = 0$ since A_k is independent of $\mathcal{F}_{k-1}$.

Let us show that condition (3.1) holds for S_n. Since A_k is independent of $\mathcal{F}_{k-1}$, we have

$$\mathbb{E}^{\mathcal{F}_{k-1}}\, d_{n,k}^{\otimes 2} = \mathbb{E}\, d_{n,k}^{\otimes 2} = \frac{1}{n}\left(e^{\mathbb{E}\, A\frac{k-1}{n}}\right)^{\otimes 2} \mathbb{E}(A - \mathbb{E}\, A)^{\otimes 2} \left(e^{\mathbb{E}\, A\frac{n-k}{n}}\right)^{\otimes 2}.$$

As $n \to \infty$, $k/n \to s$, $\frac{1}{n} \to ds$, the sums of these expectations converge to Σ and convergence (3.1) holds.

Let us show that conditional Lindeberg condition (3.2) holds for S_n. Let $\rho > 0$ be the constant such that $\|A\| \leqslant \rho$. Hence $\|e^A\| \leqslant e^\rho$, and

$$\|d_{n,k}\| \leqslant \frac{2\rho}{\sqrt{n}} e^{\rho\frac{n-1}{n}}.$$

Thus for each $\varepsilon > 0$ and large enough n we have $\{\|d_{n,k}\| > \varepsilon\} = \varnothing$, and (3.2) holds.

It remains to show that $\|\xi_n x - S_n x\| \xrightarrow[n\to\infty]{\mathrm{P}} 0$. Since by Lemma 3.1 for $k = n$ we have

$$\sqrt{n}\left\|(\mathbb{E}\, e^{A_1/n} \dots e^{A_n/n} - e^{\mathbb{E}\, A})x\right\| \xrightarrow[n\to\infty]{} 0,$$

it is sufficient to show that

$$\|S_n x - \xi_n' x\| \xrightarrow[n\to\infty]{\mathrm{P}} 0,$$

where

$$\xi_n' := \sqrt{n}\left(e^{A_1/n}\dots e^{A_n/n} - \left(\mathbb{E}\, e^{A/n}\right)^n\right) = \sqrt{n}\sum_{k=1}^{n} e^{A_1/n}\dots e^{A_{k-1}/n}\left(e^{A_k/n} - \mathbb{E}\, e^{A/n}\right)\left(\mathbb{E}\, e^{A/n}\right)^{n-k}.$$

Applying the Taylor formula for $e^{A_k/n} - \mathbb{E}\, e^{A/n}$, we obtain

$$\xi_n' = S_n' + R_n,$$

where

$$S_n' := \frac{1}{\sqrt{n}}\sum_{k=1}^{n} e^{A_1/n}\dots e^{A_{k-1}/n}\left(A_k - \mathbb{E}\, A\right)\left(\mathbb{E}\, e^{A/n}\right)^{n-k}$$
$$\|R_n\| = \sqrt{n}\cdot n\cdot e^{\rho}\cdot O(1/n^2) = O(1/\sqrt{n}).$$

Now it remains to show that $\|S_n x - S_n' x\| \xrightarrow[n\to\infty]{\mathrm{P}} 0$. By the Markov inequality it is sufficient to show that

$$\mathbb{E}\,\|S_n x - S_n' x\|^2 \xrightarrow[n\to\infty]{} 0.$$

We denote the summands in S_n' by $d_{n,k}'$ and we find

$$\mathbb{E}\,\|S_n x - S_n' x\|^2 = \sum_{k,\ell}\mathbb{E}\langle d_{n,k}x - d_{n,k}'x, d_{n,\ell}x - d_{n,\ell}'x\rangle = \sum_{k=1}^{n}\mathbb{E}\left\|d_{n,k}x - d_{n,k}'x\right\|^2,$$

where in the last equality we used that for $k > \ell$ by the law of total probability

$$\mathbb{E}\langle d_{n,k}x - d_{n,k}'x, d_{n,\ell}x - d_{n,\ell}'x\rangle = \mathbb{E}\left\langle \mathbb{E}^{\mathcal{F}_{k-1}}(d_{n,k}x - d_{n,k}'x), d_{n,\ell}x - d_{n,\ell}'x\right\rangle = 0,$$

since $\mathbb{E}^{\mathcal{F}_{k-1}}\, d_{n,k} = \mathbb{E}^{\mathcal{F}_{k-1}}\, d_{n,k}' = 0$. Finally, in order to complete the proof it is sufficient to show that

$$\mathbb{E}\left\|d_{n,k}x - d_{n,k}'x\right\|^2 = O\left(1/n^2\right),$$

where the constant is independent of $k = k(n)$.

We denote

$$M_k := e^{A_1/n}\dots e^{A_k/n}x - \left(\mathbb{E}\, e^{A/n}\right)^k x.$$

We observe that up to an inessential constant depending on ρ we have

$$\sqrt{n}\left\|d_{n,k}x - d_{n,k}'x\right\| \leqslant \|M_{k-1}\| + \left\|\left(\mathbb{E}\, e^{A/n}\right)^{k-1}x - e^{\mathbb{E}\, A\frac{k-1}{n}}x\right\| + \left\|\left(\mathbb{E}\, e^{A/n}\right)^{n-k}x - e^{\mathbb{E}\, A\frac{n-k}{n}}x\right\|.$$

Here the last two summands are $O(1/n)$ by Lemma 3.1. Then

$$n\left\|d_{n,k}x - d_{n,k}'x\right\|^2 \leqslant \|M_{k-1}\|^2 + \|M_{k-1}\|\cdot O(1/n) + O(1/n^2).$$

Similarly to [10], for integer $0 \leqslant m \leqslant k$ we denote $[k] := \{1,\dots,k\}$, while $\binom{[k]}{m}$ stands for the family of m-element subsets of $[k]$. For $\{i_1 < \dots < i_m\} \in \binom{[k]}{m}$ we let

$$F_{k,\{i_1,\dots,i_m\}} := (\mathbb{E}\, e^{A/n})^{i_1-1}\left(e^{A_{i_1}/n} - \mathbb{E}\, e^{A/n}\right)(\mathbb{E}\, e^{A/n})^{i_2-i_1-1}\dots\left(e^{A_{i_m}/n} - \mathbb{E}\, e^{A/n}\right)(\mathbb{E}\, e^{A/n})^{k-i_m}.$$

Denoting

$$D_{k,m} := \sum_{\substack{P\subset[k]\\ \max P = m}} F_{k,P}x,$$

similarly to [10] we obtain

$$M_k = \sum_{m=1}^{k} D_{k,m}$$

is the martingale with respect to $\mathcal{F}_k = \sigma(A_1, \ldots, A_k)$. By [10, Lemma 3.3] we have

$$\|F_{k,P}\| \leqslant \left(\frac{2\rho}{n}\right)^{|P|} e^{k\rho/n}.$$

Hence, up to $\|x\|$,

$$\|D_{k,m}\| \leqslant \sum_{j=1}^{m} \binom{m-1}{j-1} \left(\frac{2\rho}{n}\right)^{j} e^{\rho} = \frac{2\rho}{n} e^{\rho} \left(1 + \frac{2\rho}{n}\right)^{m-1} \leqslant \frac{2\rho}{n} e^{3\rho}.$$

Since $\mathcal{H}$ is 2-smooth, we can apply [31, Thm. 4.52]; see also Burkholder–type Theorem 4.1 in [10, 4.1], for our arguing. Up to some constant independent of k we obtain

$$\mathbb{E}\|M_k\| \leqslant \sqrt{n \cdot \left(\frac{2\rho}{n} e^{3\rho}\right)^2} = O(1/\sqrt{n}) \quad \text{and} \quad \mathbb{E}\|M_k\|^2 \leqslant n \cdot \left(\frac{2\rho}{n} e^{3\rho}\right)^2 = O(1/n).$$

Thus,

$$\left\|d_{n,k}x - d'_{n,k}x\right\|^2 = O\left(1/n^2\right),$$

and this completes the proof. □

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