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GLOBAL AND BLOW-UP SOLUTIONS FOR A PARABOLIC EQUATION WITH NONLINEAR MEMORY UNDER NONLINEAR NONLOCAL BOUNDARY CONDITION

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Abstract. In this paper we consider parabolic equation with nonlinear memory and absorption

$$u_t = \Delta u + a \int_0^t u^q(x, \tau) d\tau - bu^m, \quad x \in \Omega, \quad t > 0,$$

under nonlinear nonlocal boundary condition

$$u(x, t) = \int_{\Omega} k(x, y, t) u^l(y, t) dy, \quad x \in \partial\Omega, \quad t > 0,$$

and nonnegative continuous initial data. Here a, b, q, m, l are positive numbers, Ω is a bounded domain in \mathbb{R}^N , $N \geq 1$, with smooth boundary $\partial\Omega$, $k(x, y, t)$ is a nonnegative continuous function defined for $x \in \partial\Omega$, $y \in \bar{\Omega}$ and $t \geq 0$. We prove that each solution of the problem is global if $\max(q, l) \leq 1$ or $\max(q, l) > 1$ and $l < (m + 1)/2$, $q \leq m$. If $l > \max\{1, (p + 1)/2\}$ and the function $k(x, y, t)$ is positive for small t , the solutions blow up in finite time for large enough initial data. The obtained results improve previously established conditions for the existence and absence of global solutions.

Keywords: Parabolic equation, nonlinear memory, nonlocal boundary condition, global existence.

Mathematics Subject Classification: 35K20, 35K58, 35K61

1. INTRODUCTION

In this paper we consider the parabolic equation with nonlinear memory and absorption

$$u_t = \Delta u + a \int_0^t u^q(x, \tau) d\tau - bu^m, \quad x \in \Omega, \quad t > 0, \quad (1.1)$$

under the nonlinear nonlocal boundary condition

$$u(x, t) = \int_{\Omega} k(x, y, t) u^l(y, t) dy, \quad x \in \partial\Omega, \quad t > 0, \quad (1.2)$$

and the initial data

$$u(x, 0) = u_0(x), \quad x \in \Omega, \quad (1.3)$$

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where a, b, q, m, l are positive numbers, Ω is a bounded domain in \mathbb{R}^N for $N \geq 1$ with a smooth boundary $\partial\Omega$. Here $k(x, y, t)$ is a nonnegative continuous function defined for $x \in \partial\Omega$, $y \in \overline{\Omega}$ and $t \geq 0$. The initial data $u_0(x)$ is a nonnegative continuous function satisfying the boundary condition at $t = 0$.

Various phenomena in the natural sciences and engineering lead to the nonclassical mathematical models subject to nonlocal boundary conditions. For global existence and blow-up of solutions for parabolic equations and systems with nonlocal boundary conditions we refer to [1] – [15] and the references therein. A blow-up problem for parabolic equations with nonlocal boundary condition (1.2) was studied in [16]–[25]. The problem (1.1)–(1.3) with $a = 0$ was investigated in [18], [19].

A blow-up problem for (1.1) – (1.3) with $a = 1$ and $k(x, y, t) = f(x, y)$ was considered in [21]. To formulate the results of [21] we introduce some notation. Let λ_1 be the lowest eigenvalue of the following problem

$$-\Delta\varphi = \lambda\varphi, \quad x \in \Omega, \quad \varphi = 0, \quad x \in \partial\Omega, \quad (1.4)$$

and the associated eigenfunction $\varphi = \varphi(x)$ is fixed by the condition

$$\int_{\Omega} \varphi(x) dx = 1.$$

We denote

$$L = \max_{\overline{\Omega}} \varphi(x), \quad m_0 = \min_{\partial\Omega \times \overline{\Omega}} f(x, y).$$

The main global existence and blow-up results of [21] are as follows.

Theorem 1.1. *Let $q \leq m$ and $l \leq 1$. Then the problem (1.1)–(1.3) has global solutions for each $f(x, y)$ and $u_0(x)$.*

Theorem 1.2. *Let $q > m \geq 1$. Then a solution of the problem (1.1)–(1.3) blows up in finite time for each $f(x, y)$ and $u_0(x) \not\equiv 0$.*

Theorem 1.3. *Let $l \geq m \geq q > 1$ and $m_0\lambda_1 > bL$. Then a solution of the problem (1.1)–(1.3) blows up in finite time if the initial data $u_0(x)$ satisfies $\int_{\Omega} u_0(x)\varphi(x) dx \gg 1$.*

The aim of this paper is to improve global existence and blow-up results of [21].

2. GLOBAL EXISTENCE AND BLOW-UP

We begin with the definition of the supersolution, subsolution and solution of (1.1)–(1.3). Let

$$Q_T = \Omega \times (0, T), \quad S_T = \partial\Omega \times (0, T), \quad \Gamma_T = S_T \cup \overline{\Omega} \times \{0\}, \quad T > 0.$$

Definition 2.1. *We say that a nonnegative function $u(x, t) \in C^{2,1}(Q_T) \cap C(Q_T \cup \Gamma_T)$ is a supersolution of (1.1)–(1.3) in Q_T if*

$$u_t \geq \Delta u + a \int_0^t u^q(x, \tau) d\tau - bu^m, \quad (x, t) \in Q_T, \quad (2.1)$$

$$u(x, t) \geq \int_{\Omega} k(x, y, t) u^l(y, t) dy, \quad x \in \partial\Omega, \quad 0 < t < T, \quad (2.2)$$

$$u(x, 0) \geq u_0(x), \quad x \in \Omega, \quad (2.3)$$

and $u(x, t) \in C^{2,1}(Q_T) \cap C(Q_T \cup \Gamma_T)$ is a subsolution of (1.1)–(1.3) in Q_T if $u \geq 0$ and the reversed inequalities hold in (2.1)–(2.3). We say that $u(x, t)$ is a solution of the problem (1.1)–(1.3) in Q_T if $u(x, t)$ is both a subsolution and a supersolution of (1.1)–(1.3) in Q_T .

To prove the main results, we use the comparison principle which can be established as in [19].

Theorem 2.1. *Let \bar{u} and \underline{u} be a supersolution and a subsolution of the problem (1.1)–(1.3) in Q_T , respectively. Suppose that $\underline{u}(x, t) > 0$ or $\bar{u}(x, t) > 0$ in $Q_T \cup \Gamma_T$ if $\min(q, l) < 1$. Then $\bar{u}(x, t) \geq \underline{u}(x, t)$ in $Q_T \cup \Gamma_T$.*

The proof of global existence of solutions relies on the continuation principle and the construction of a supersolution.

Theorem 2.2. *Let at least one from the following conditions hold:*

- a) $\max(q, l) \leq 1$;
- b) $\max(q, l) > 1$ and $l < (m + 1)/2$, $q \leq m$.

Then each solution of (1.1)–(1.3) is global.

Proof. In order to prove global existence of solutions we construct a suitable explicit supersolution of (1.1)–(1.3) in Q_T for each positive T .

Suppose first that $\max(q, l) \leq 1$ or $l \leq 1$, $1 < q \leq m$. We construct a supersolution as in [16], [21]. Since $k(x, y, t)$ is a continuous function, there exists a constant $K > 0$ such that

$$k(x, y, t) \leq K \quad (2.4)$$

in $\partial\Omega \times Q_T$. Let $\varphi(x)$ be the eigenfunction of the problem (1.4) corresponding the lowest eigenvalue λ_1 such that

$$K \int_{\Omega} \frac{dy}{(\varphi(y) + 1)^l} \leq 1. \quad (2.5)$$

We construct a supersolution of (1.1)–(1.3) in Q_T as

$$\bar{u}(x, t) = \frac{C \exp(\mu t)}{\varphi(x) + 1},$$

where constants C and μ are chosen to satisfy the inequalities

$$C \geq \max \left\{ \sup_{\Omega} (\varphi(x) + 1) \sup_{\Omega} (u_0(x) + 1), 1 \right\}, \quad (2.6)$$

$$\mu \geq 1 + \lambda_1 + 2 \sup_{\Omega} \frac{|\nabla \varphi|^2}{(\varphi(x) + 1)^2} + \frac{a}{q} \text{ for } q \leq 1, \quad (2.7)$$

$$\mu \geq \lambda_1 + 2 \sup_{\Omega} \frac{|\nabla \varphi|^2}{(\varphi(x) + 1)^2} + \frac{a}{bq} \text{ for } 1 < q \leq m. \quad (2.8)$$

By (2.5)–(2.8) we easily obtain

$$\begin{aligned} \bar{u}_t - \Delta \bar{u} - a \int_0^t u^q(x, \tau) d\tau + b \bar{u}^m &= \mu \bar{u} - \left(\frac{\lambda_1 \varphi}{\varphi(x) + 1} + 2 \frac{|\nabla \varphi|^2}{(\varphi(x) + 1)^2} \right) \bar{u} \\ &\quad - \frac{a C^q \exp(q \mu t)}{q \mu (\varphi(x) + 1)^q} + \frac{a C^q}{q \mu (\varphi(x) + 1)^q} \\ &\quad + \frac{b C^m \exp(m \mu t)}{(\varphi(x) + 1)^m} \geq 0 \end{aligned} \quad (2.9)$$

for $(x, t) \in Q_T$,

$$\bar{u}(x, t) = C \exp(\mu t) \geq C^l \exp(l\mu t) K \int_{\Omega} \frac{dy}{(\varphi(y) + 1)^l} \geq \int_{\Omega} k(x, y, t) \bar{u}^l(y, t) dy \quad (2.10)$$

for $(x, t) \in S_T$ and

$$\bar{u}(x, 0) \geq u_0(x) \quad (2.11)$$

for $x \in \Omega$. By virtue of (2.9)–(2.11) the solution of (1.1)–(1.3) exists globally.

Suppose that $1 < l < (m + 1)/2$, $q \leq m$. To construct a supersolution we use the change of variables in a neighborhood of $\partial\Omega$ as in [26]. Let $\bar{x} \in \partial\Omega$ and $\hat{n}(\bar{x})$ be the inward unit normal to $\partial\Omega$ at the point \bar{x} . Since $\partial\Omega$ is smooth, it is well known that there exists $\delta > 0$ such that the mapping $\psi : \partial\Omega \times [0, \delta] \rightarrow \mathbb{R}^n$ given by $\psi(\bar{x}, s) = \bar{x} + s\hat{n}(\bar{x})$ defines new coordinates (\bar{x}, s) in a neighborhood of $\partial\Omega$ in $\bar{\Omega}$. A straightforward computation shows that, in these coordinates, Δ applied to a function $g(\bar{x}, s) = g(s)$, which is independent of the variable \bar{x} , evaluated at a point (\bar{x}, s) is given by

$$\Delta g(\bar{x}, s) = \frac{\partial^2 g}{\partial s^2}(\bar{x}, s) - \sum_{j=1}^{n-1} \frac{H_j(\bar{x})}{1 - sH_j(\bar{x})} \frac{\partial g}{\partial s}(\bar{x}, s), \quad (2.12)$$

where $H_j(\bar{x})$ for $j = 1, \dots, n-1$, denote the principal curvatures of $\partial\Omega$ at \bar{x} . For $0 \leq s \leq \delta$ and small δ we have

$$\left| \sum_{j=1}^{n-1} \frac{H_j(\bar{x})}{1 - sH_j(\bar{x})} \right| \leq \bar{c}. \quad (2.13)$$

Let

$$\begin{aligned} 0 < \varepsilon < \omega < \min(\delta, 1), \quad \max\left(\frac{1}{l}, \frac{2}{m-1}\right) < \beta < \frac{1}{l-1}, \\ 0 < \gamma < \frac{\beta}{2}, \quad A \geq \sup_{\Omega} u_0(x), \quad r > \frac{a}{bq}. \end{aligned}$$

We modify the supersolution from [18]. For points in $Q_{\delta, T} = \partial\Omega \times [0, \delta] \times [0, T]$ with the coordinates (\bar{x}, s, t) we define

$$\bar{v}(x, t) = \bar{v}((\bar{x}, s), t) = \left([(s + \varepsilon)^{-\gamma} - \omega^{-\gamma}]_+^{\frac{\beta}{\gamma}} + A \right) \exp(rt), \quad (2.14)$$

where $s_+ = \max(s, 0)$. For points in $\bar{Q}_T \setminus Q_{\delta, T}$ we let $\bar{v}(x, t) = A \exp(rt)$. We are going to show that $\bar{v}(x, t)$ is the supersolution of (1.1)–(1.3) in the set Q_T . It is easy to verify that

$$\left| \frac{\partial \bar{v}}{\partial s} \right| \leq \beta \min \left([D(s)]^{\frac{\gamma+1}{\gamma}} [(s + \varepsilon)^{-\gamma} - \omega^{-\gamma}]_+^{\frac{\beta+1}{\gamma}}, (s + \varepsilon)^{-(\beta+1)} \right) \exp(rt), \quad (2.15)$$

$$\left| \frac{\partial^2 \bar{v}}{\partial s^2} \right| \leq \beta(\beta + 1) \min \left([D(s)]^{\frac{2(\gamma+1)}{\gamma}} [(s + \varepsilon)^{-\gamma} - \omega^{-\gamma}]_+^{\frac{\beta+2}{\gamma}}, (s + \varepsilon)^{-(\beta+2)} \right) \exp(rt), \quad (2.16)$$

where

$$D(s) = \frac{(s + \varepsilon)^{-\gamma}}{(s + \varepsilon)^{-\gamma} - \omega^{-\gamma}}.$$

Then $D'(s) > 0$ and for each $\bar{\varepsilon} > 0$

$$1 \leq D(s) \leq 1 + \bar{\varepsilon}, \quad 0 < s \leq \bar{s}, \quad (2.17)$$

where

$$\bar{s} = [\bar{\varepsilon}/(1 + \bar{\varepsilon})]^{1/\gamma} \omega - \varepsilon, \quad \varepsilon < [\bar{\varepsilon}/(1 + \bar{\varepsilon})]^{1/\gamma} \omega.$$

We denote

$$Lv \equiv v_t - \Delta v - a \int_0^t v^q(x, \tau) d\tau + bv^m. \quad (2.18)$$

By (2.12)–(2.18) we can choose $\bar{\varepsilon}$ small and A large so that in $Q_{\bar{s}, T}$ the inequalities hold

$$\begin{aligned} L\bar{v} \geq & \left\{ r \left([(s + \varepsilon)^{-\gamma} - \omega^{-\gamma}]_+^{\frac{\beta}{\gamma}} + A \right) + b \left([(s + \varepsilon)^{-\gamma} - \omega^{-\gamma}]_+^{\frac{\beta}{\gamma}} + A \right)^m \exp[r(m-1)t] \right. \\ & - \beta(\beta+1) [D(s)]^{\frac{2(\gamma+1)}{\gamma}} [(s + \varepsilon)^{-\gamma} - \omega^{-\gamma}]_+^{\frac{\beta+2}{\gamma}} + \beta \bar{c} [D(s)]^{\frac{\gamma+1}{\gamma}} [(s + \varepsilon)^{-\gamma} - \omega^{-\gamma}]_+^{\frac{\beta+1}{\gamma}} \\ & \left. - \frac{a}{rq} \left([(s + \varepsilon)^{-\gamma} - \omega^{-\gamma}]_+^{\frac{\beta}{\gamma}} + A \right)^q \exp[r(q-1)t] \right\} \exp(rt) \geq 0. \end{aligned}$$

Let $s \in [\bar{s}, \delta]$. It follows from (2.12)–(2.16) that

$$|\Delta \bar{v}| \leq \left\{ \beta(\beta+1) \omega^{-(\beta+2)} \left(\frac{1+\bar{\varepsilon}}{\bar{\varepsilon}} \right)^{\frac{\beta+2}{\gamma}} + \beta \bar{c} \omega^{-(\beta+1)} \left(\frac{1+\bar{\varepsilon}}{\bar{\varepsilon}} \right)^{\frac{\beta+1}{\gamma}} \right\} \exp(rt)$$

and $L\bar{v} \geq 0$ for large values of A . It is obvious that

$$L\bar{v} = rA \exp(rt) - \frac{aA^q}{rq} [\exp(rqt) - 1] + bA^m \exp(rmt) \geq 0$$

in $\overline{Q_T} \setminus Q_{\delta, T}$ for $A \geq 1$.

Let us prove the inequality

$$\bar{v}(\bar{x}, 0, t) \geq \int_{\Omega} K \bar{v}^l(\bar{x}, s, t) dy, \quad (x, t) \in S_T \quad (2.19)$$

for a suitable choice of ε . To estimate the integral I in the right hand side of (2.19), we use the change of variables in a neighborhood of $\partial\Omega$ as above. Let

$$\bar{J} = \sup_{0 < s < \delta} \int_{\partial\Omega} |J(\bar{y}, s)| d\bar{y},$$

where $J(\bar{y}, s)$ is Jacobian of the change of variables. Then we have

$$\begin{aligned} I & \leq 2^{l-1} K \left\{ \int_{\Omega} [(s + \varepsilon)^{-\gamma} - \omega^{-\gamma}]_+^{\frac{\beta l}{\gamma}} dy + A^l |\Omega| \right\} \exp(rlt) \\ & \leq 2^{l-1} K \left\{ \bar{J} \int_0^{\omega^{-\varepsilon}} [(s + \varepsilon)^{-\gamma} - \omega^{-\gamma}]_+^{\frac{\beta l}{\gamma}} ds + A^l |\Omega| \right\} \exp(rlt) \\ & \leq 2^{l-1} K \left\{ \frac{\bar{J}}{\beta l - 1} [\varepsilon^{-(\beta l - 1)} - \omega^{-(\beta l - 1)}] + A^l |\Omega| \right\} \exp(rlt), \end{aligned}$$

where the constant K was defined in (2.4). On the other hand, since

$$\bar{v}(\bar{x}, 0, t) = \left([\varepsilon^{-\gamma} - \omega^{-\gamma}]_+^{\frac{\beta}{\gamma}} + A \right) \exp(rt),$$

the inequality (2.19) holds if ε is small enough. At last,

$$u(x, 0) \leq \bar{v}(x, 0) \quad \text{in } \Omega.$$

Hence, by Theorem 2.1 we get

$$u(x, t) \leq \bar{v}(x, t) \quad \text{in } \overline{Q_T}.$$

The proof is complete. \square

Now we prove finite time blow-up result. We shall suppose that

$$k(x, y, t) \geq k_0 > 0, \quad x \in \partial\Omega, \quad y \in \Omega, \quad 0 < t < \beta \quad (2.20)$$

for some positive k_0 and β .

Theorem 2.3. *Let $l > \max\{1, (m+1)/2\}$ and (2.20) hold. Then the solutions of problem (1.1)–(1.3) blow up in finite time for large enough initial data.*

Proof. We observe that each solution of (1.1) – (1.3) is a supersolution of the same problem with $a = 0$. Then by Theorem 2.2 and Theorem 2.6 of [18] the solutions of the problem (1.1)–(1.3) blow up in finite time for large enough initial data. \square

Remark 2.1. *In the case $q \leq m$, $l = (m+1)/2 > 1$ the global existence and blow-up results depend on b and $k(x, y, t)$.*

Remark 2.2. *We note that the proof of Theorem 1.2 in [21] is valid also for $q > 1 > m$. So, a solution of the problem (1.1)–(1.3) blows up in finite time for each $u_0(x) \not\equiv 0$ if $q > \max(m, 1)$.*

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