GLOBAL AND BLOW–UP SOLUTIONS FOR A PARABOLIC EQUATION WITH NONLINEAR MEMORY UNDER NONLINEAR NONLOCAL BOUNDARY CONDITION

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Abstract. In this paper we consider parabolic equation with nonlinear memory and absorption

$$u_t = \Delta u + a \int\limits_0^t u^q(x,\tau)\,d\tau - bu^m, \quad x \in \Omega, \quad t > 0,$$

under nonlinear nonlocal boundary condition

$$u(x,t) = \int\limits_\Omega k(x,y,t)u^l(y,t)\,dy, \quad x \in \partial\Omega, \quad t > 0,$$

and nonnegative continuous initial data. Here a, b, q, m, l are positive numbers, Ω is a bounded domain in $\mathbb{R}^N$, $N \geqslant 1$, with smooth boundary $\partial\Omega$, $k(x,y,t)$ is a nonnegative continuous function defined for $x \in \partial\Omega$, $y \in \overline{\Omega}$ and $t \geqslant 0$. We prove that each solution of the problem is global if $\max(q,l) \leqslant 1$ or $\max(q,l) > 1$ and $l < (m+1)/2$, $q \leqslant m$. If $l > \max\{1, (p+1)/2\}$ and the function $k(x,y,t)$ is positive for small t, the solutions blow up in finite time for large enough initial data. The obtained results improve previously established conditions for the existence and absence of global solutions.

Keywords: Parabolic equation, nonlinear memory, nonlocal boundary condition, global existence.

Mathematics Subject Classification: 35K20, 35K58, 35K61

1. Introduction

In this paper we consider the parabolic equation with nonlinear memory and absorption

$$u_t = \Delta u + a \int\limits_0^t u^q(x,\tau)\,d\tau - bu^m, \quad x \in \Omega, \quad t > 0, \tag{1.1}$$

under the nonlinear nonlocal boundary condition

$$u(x,t) = \int\limits_\Omega k(x,y,t)u^l(y,t)\,dy, \quad x \in \partial\Omega, \quad t > 0, \tag{1.2}$$

A.L. Gladkov, Global and blow–up solutions for a parabolic equation with nonlinear memory under nonlinear nonlocal boundary condition.

This work is supported by the State Program of Fundamental Research of Belarus (grant 1.2.02.2).

Submitted February 20, 2024.

and the initial data

$$u(x,0) = u_0(x), \quad x \in \Omega, \tag{1.3}$$

where a, b,q,m,l are positive numbers, Ω is a bounded domain in $\mathbb{R}^N$ for $N \geqslant 1$ with a smooth boundary $\partial\Omega$. Here $k(x,y,t)$ is a nonnegative continuous function defined for $x \in \partial\Omega$, $y \in \overline{\Omega}$ and $t \geqslant 0$. The initial data $u_0(x)$ is a nonnegative continuous function satisfying the boundary condition at $t = 0$.

Various phenomena in the natural sciences and engineering lead to the nonclassical mathematical models subject to nonlocal boundary conditions. For global existence and blow–up of solutions for parabolic equations and systems with nonlocal boundary conditions we refer to [1] – [15] and the references therein. A blow–up problem for parabolic equations with nonlocal boundary condition (1.2) was studied in [16]–[25]. The problem (1.1)–(1.3) with $a = 0$ was investigated in [18], [19].

A blow–up problem for (1.1) – (1.3) with $a = 1$ and $k(x,y,t) = f(x,y)$ was considered in [21]. To formulate the results of [21] we introduce some notation. Let λ_1 be the lowest eigenvalue of the following problem

$$-\Delta\varphi = \lambda\varphi, \quad x \in \Omega, \qquad \varphi = 0, \quad x \in \partial\Omega, \tag{1.4}$$

and the associated eigenfunction $\varphi = \varphi(x)$ is fixed by the condition

$$\int_\Omega \varphi(x)\,dx = 1.$$

We denote

$$L = \max_{\overline{\Omega}} \varphi(x), \qquad m_0 = \min_{\partial\Omega\times\overline{\Omega}} f(x,y).$$

The main global existence and blow–up results of [21] are as follows.

Theorem 1.1. *Let $q \leqslant m$ and $l \leqslant 1$. Then the problem (1.1)–(1.3) has global solutions for each $f(x,y)$ and $u_0(x)$.*

Theorem 1.2. *Let $q > m \geqslant 1$. Then a solution of the problem (1.1)–(1.3) blows up in finite time for each $f(x,y)$ and $u_0(x) \not\equiv 0$.*

Theorem 1.3. *Let $l \geqslant m \geqslant q > 1$ and $m_0\lambda_1 > bL$. Then a solution of the problem (1.1)–(1.3) blows up in finite time if the initial data $u_0(x)$ satisfies $\int_\Omega u_0(x)\varphi(x)\,dx \gg 1$.*

The aim of this paper is to improve global existence and blow–up results of [21].

2. Global existence and blow–up

We begin with the definition of the supersolution, subsolution and solution of (1.1)–(1.3). Let

$$Q_T = \Omega \times (0,T), \qquad S_T = \partial\Omega \times (0,T), \qquad \Gamma_T = S_T \cup \overline{\Omega} \times \{0\}, \quad T > 0.$$

Definition 2.1. *We say that a nonnegative function $u(x,t) \in C^{2,1}(Q_T) \cap C(Q_T \cup \Gamma_T)$ is a supersolution of (1.1)–(1.3) in Q_T if*

$$u_t \geqslant \Delta u + a\int_0^t u^q(x,\tau)\,d\tau - bu^m, \quad (x,t) \in Q_T, \tag{2.1}$$

$$u(x,t) \geqslant \int_\Omega k(x,y,t)u^l(y,t)\,dy, \quad x \in \partial\Omega, \quad 0 < t < T, \tag{2.2}$$

$$u(x,0) \geqslant u_0(x), \quad x \in \Omega, \tag{2.3}$$

and $u(x,t) \in C^{2,1}(Q_T) \cap C(Q_T \cup \Gamma_T)$ *is a subsolution of (1.1)–(1.3) in* Q_T *if* $u \geqslant 0$ *and the reversed inequalities hold in (2.1)–(2.3). We say that* $u(x,t)$ *is a solution of the problem (1.1)–(1.3) in* Q_T *if* $u(x,t)$ *is both a subsolution and a supersolution of (1.1)–(1.3) in* Q_T.

To prove the main results, we use the comparison principle which can be established as in [19].

Theorem 2.1. *Let* $\overline{u}$ *and* $\underline{u}$ *be a supersolution and a subsolution of the problem (1.1)–(1.3) in* Q_T*, respectively. Suppose that* $\underline{u}(x,t) > 0$ *or* $\overline{u}(x,t) > 0$ *in* $Q_T \cup \Gamma_T$ *if* $\min(q,l) < 1$*. Then* $\overline{u}(x,t) \geqslant \underline{u}(x,t)$ *in* $Q_T \cup \Gamma_T$.

The proof of global existence of solutions relies on the continuation principle and the construction of a supersolution.

Theorem 2.2. *Let at least one from the following conditions hold:*

a) $\max(q,l) \leqslant 1$;
b) $\max(q,l) > 1$ *and* $l < (m+1)/2$, $q \leqslant m$.

Then each solution of (1.1)–(1.3) is global.

Доказательство. In order to prove global existence of solutions we construct a suitable explicit supersolution of (1.1)–(1.3) in Q_T for each positive T.

Suppose first that $\max(q,l) \leqslant 1$ or $l \leqslant 1$, $1 < q \leqslant m$. We construct a supersolution as in [16], [21]. Since $k(x,y,t)$ is a continuous function, there exists a constant $K > 0$ such that

$$k(x,y,t) \leqslant K \tag{2.4}$$

in $\partial\Omega \times Q_T$. Let $\varphi(x)$ be the eigenfunction of the problem (1.4) corresponding the lowest eigenvalue λ_1 such that

$$K \int_{\Omega} \frac{dy}{(\varphi(y)+1)^l} \leqslant 1. \tag{2.5}$$

We construct a supersolution of (1.1)–(1.3) in Q_T as

$$\overline{u}(x,t) = \frac{C\exp(\mu t)}{\varphi(x)+1},$$

where constants C and μ are chosen to satisfy the inequalities

$$C \geqslant \max\left\{\sup_{\Omega}(\varphi(x)+1)\sup_{\Omega}(u_0(x)+1), 1\right\}, \tag{2.6}$$

$$\mu \geqslant 1 + \lambda_1 + 2\sup_{\Omega}\frac{|\nabla\varphi|^2}{(\varphi(x)+1)^2} + \frac{a}{q} \quad \text{for } \ q \leqslant 1, \tag{2.7}$$

$$\mu \geqslant \lambda_1 + 2\sup_{\Omega}\frac{|\nabla\varphi|^2}{(\varphi(x)+1)^2} + \frac{a}{bq} \quad \text{for } \ 1 < q \leqslant m. \tag{2.8}$$

By (2.5)–(2.8) we easily obtain

$$\begin{aligned} \overline{u}_t - \Delta\overline{u} - a\int_0^t u^q(x,\tau)\,d\tau + b\overline{u}^m =& \mu\overline{u} - \left(\frac{\lambda_1\varphi}{\varphi(x)+1} + 2\frac{|\nabla\varphi|^2}{(\varphi(x)+1)^2}\right)\overline{u} \\ &- \frac{aC^q\exp(q\mu t)}{q\mu(\varphi(x)+1)^q} + \frac{aC^q}{q\mu(\varphi(x)+1)^q} \\ &+ \frac{bC^m\exp(m\mu t)}{(\varphi(x)+1)^m} \geqslant 0 \end{aligned} \tag{2.9}$$

for $(x,t)\in Q_T$,

$$\overline{u}(x,t)=C\exp(\mu t)\geqslant C^l\exp(l\mu t)K\int\limits_\Omega\frac{dy}{(\varphi(y)+1)^l}\geqslant\int\limits_\Omega k(x,y,t)\overline{u}^l(y,t)\,dy \tag{2.10}$$

for $(x,t)\in S_T$ and

$$\overline{u}(x,0)\geqslant u_0(x) \tag{2.11}$$

for $x\in\Omega$. By virtue of (2.9)–(2.11) the solution of (1.1)–(1.3) exists globally.

Suppose that $1<l<(m+1)/2$, $q\leqslant m$. To construct a supersolution we use the change of variables in a neighborhood of $\partial\Omega$ as in [26]. Let $\overline{x}\in\partial\Omega$ and $\widehat{n}(\overline{x})$ be the inward unit normal to $\partial\Omega$ at the point $\overline{x}$. Since $\partial\Omega$ is smooth, it is well known that there exists $\delta>0$ such that the mapping $\psi:\partial\Omega\times[0,\delta]\to\mathbb{R}^n$ given by $\psi(\overline{x},s)=\overline{x}+s\widehat{n}(\overline{x})$ defines new coordinates $(\overline{x},s)$ in a neighborhood of $\partial\Omega$ in $\overline{\Omega}$. A straightforward computation shows that, in these coordinates, Δ applied to a function $g(\overline{x},s)=g(s)$, which is independent of the variable $\overline{x}$, evaluated at a point $(\overline{x},s)$ is given by

$$\Delta g(\overline{x},s)=\frac{\partial^2 g}{\partial s^2}(\overline{x},s)-\sum_{j=1}^{n-1}\frac{H_j(\overline{x})}{1-sH_j(\overline{x})}\frac{\partial g}{\partial s}(\overline{x},s), \tag{2.12}$$

where $H_j(\overline{x})$ for $j=1,...,n-1$, denote the principal curvatures of $\partial\Omega$ at $\overline{x}$. For $0\leqslant s\leqslant\delta$ and small δ we have

$$\left|\sum_{j=1}^{n-1}\frac{H_j(\overline{x})}{1-sH_j(\overline{x})}\right|\leqslant\overline{c}. \tag{2.13}$$

Let

$$0<\varepsilon<\omega<\min(\delta,1),\qquad \max\left(\frac{1}{l},\frac{2}{m-1}\right)<\beta<\frac{1}{l-1},$$
$$0<\gamma<\frac{\beta}{2},\qquad A\geqslant\sup_\Omega u_0(x),\qquad r>\frac{a}{bq}.$$

We modify the supersolution from [18]. For points in $Q_{\delta,T}=\partial\Omega\times[0,\delta]\times[0,T]$ with the coordinates $(\overline{x},s,t)$ we define

$$\overline{v}(x,t)=\overline{v}((\overline{x},s),t)=\left(\left[(s+\varepsilon)^{-\gamma}-\omega^{-\gamma}\right]_+^{\frac{\beta}{\gamma}}+A\right)\exp(rt), \tag{2.14}$$

where $s_+=\max(s,0)$. For points in $\overline{Q_T}\setminus Q_{\delta,T}$ we let $\overline{v}(x,t)=A\exp(rt)$. We are going to show that $\overline{v}(x,t)$ is the supersolution of (1.1)–(1.3) in the set Q_T. It is easy to verify that

$$\left|\frac{\partial\overline{v}}{\partial s}\right|\leqslant\beta\min\left([D(s)]^{\frac{\gamma+1}{\gamma}}\left[(s+\varepsilon)^{-\gamma}-\omega^{-\gamma}\right]_+^{\frac{\beta+1}{\gamma}},\,(s+\varepsilon)^{-(\beta+1)}\right)\exp(rt), \tag{2.15}$$

$$\left|\frac{\partial^2\overline{v}}{\partial s^2}\right|\leqslant\beta(\beta+1)\min\left([D(s)]^{\frac{2(\gamma+1)}{\gamma}}\left[(s+\varepsilon)^{-\gamma}-\omega^{-\gamma}\right]_+^{\frac{\beta+2}{\gamma}},\,(s+\varepsilon)^{-(\beta+2)}\right)\exp(rt), \tag{2.16}$$

where

$$D(s)=\frac{(s+\varepsilon)^{-\gamma}}{(s+\varepsilon)^{-\gamma}-\omega^{-\gamma}}.$$

Then $D'(s)>0$ and for each $\overline{\varepsilon}>0$

$$1\leqslant D(s)\leqslant 1+\overline{\varepsilon},\qquad 0<s\leqslant\overline{s}, \tag{2.17}$$

where

$$\overline{s}=[\overline{\varepsilon}/(1+\overline{\varepsilon})]^{1/\gamma}\omega-\varepsilon,\qquad \varepsilon<[\overline{\varepsilon}/(1+\overline{\varepsilon})]^{1/\gamma}\omega.$$

We denote

$$Lv \equiv v_t - \Delta v - a \int_0^t v^q(x,\tau)\,d\tau + bv^m. \tag{2.18}$$

By (2.12)–(2.18) we can choose $\overline{\varepsilon}$ small and A large so that in $Q_{\overline{s},T}$ the inequalities hold

$$\begin{aligned} L\overline{v} \geqslant \Big\{ & r\left(\left[(s+\varepsilon)^{-\gamma} - \omega^{-\gamma}\right]_+^{\frac{\beta}{\gamma}} + A\right) + b\left(\left[(s+\varepsilon)^{-\gamma} - \omega^{-\gamma}\right]_+^{\frac{\beta}{\gamma}} + A\right)^m \exp[r(m-1)t] \\ & - \beta(\beta+1)\left[D(s)\right]^{\frac{2(\gamma+1)}{\gamma}} \left[(s+\varepsilon)^{-\gamma} - \omega^{-\gamma}\right]_+^{\frac{\beta+2}{\gamma}} + \beta\overline{c}\left[D(s)\right]^{\frac{\gamma+1}{\gamma}} \left[(s+\varepsilon)^{-\gamma} - \omega^{-\gamma}\right]_+^{\frac{\beta+1}{\gamma}} \\ & - \frac{a}{rq}\left(\left[(s+\varepsilon)^{-\gamma} - \omega^{-\gamma}\right]_+^{\frac{\beta}{\gamma}} + A\right)^q \exp[r(q-1)t]\Big\} \exp(rt) \geqslant 0. \end{aligned}$$

Let $s \in [\overline{s},\delta]$. It follows from (2.12)–(2.16) that

$$|\Delta\overline{v}| \leqslant \left\{\beta(\beta+1)\omega^{-(\beta+2)}\left(\frac{1+\overline{\varepsilon}}{\overline{\varepsilon}}\right)^{\frac{\beta+2}{\gamma}} + \beta\overline{c}\omega^{-(\beta+1)}\left(\frac{1+\overline{\varepsilon}}{\overline{\varepsilon}}\right)^{\frac{\beta+1}{\gamma}}\right\}\exp(rt)$$

and $L\overline{v} \geqslant 0$ for large values of A. It is obvious that

$$L\overline{v} = rA\exp(rt) - \frac{aA^q}{rq}[\exp(rqt) - 1] + bA^m\exp(rmt) \geqslant 0$$

in $\overline{Q_T} \setminus Q_{\delta,T}$ for $A \geqslant 1$.

Let us prove the inequality

$$\overline{v}(\overline{x},0,t) \geqslant \int_\Omega K\overline{v}^l(\overline{x},s,t)\,dy, \quad (x,t) \in S_T \tag{2.19}$$

for a suitable choice of ε. To estimate the integral I in the right hand side of (2.19), we use the change of variables in a neighborhood of $\partial\Omega$ as above. Let

$$\overline{J} = \sup_{0<s<\delta} \int_{\partial\Omega} |J(\overline{y},s)|\,d\overline{y},$$

where $J(\overline{y},s)$ is Jacobian of the change of variables. Then we have

$$\begin{aligned} I \leqslant & 2^{l-1}K\left\{\int_\Omega \left[(s+\varepsilon)^{-\gamma} - \omega^{-\gamma}\right]_+^{\frac{\beta l}{\gamma}}\,dy + A^l|\Omega|\right\}\exp(rlt) \\ \leqslant & 2^{l-1}K\left\{\overline{J}\int_0^{\omega-\varepsilon} \left[(s+\varepsilon)^{-\gamma} - \omega^{-\gamma}\right]^{\frac{\beta l}{\gamma}}\,ds + A^l|\Omega|\right\}\exp(rlt) \\ \leqslant & 2^{l-1}K\left\{\frac{\overline{J}}{\beta l - 1}\left[\varepsilon^{-(\beta l-1)} - \omega^{-(\beta l-1)}\right] + A^l|\Omega|\right\}\exp(rlt), \end{aligned}$$

where the constant K was defined in (2.4). On the other hand, since

$$\overline{v}(\overline{x},0,t) = \left(\left[\varepsilon^{-\gamma} - \omega^{-\gamma}\right]_+^{\frac{\beta}{\gamma}} + A\right)\exp(rt),$$

the inequality (2.19) holds if ε is small enough. At last,

$$u(x,0) \leqslant \overline{v}(x,0) \quad \text{in} \quad \Omega.$$

Hence, by Theorem 2.1 we get

$$u(x,t) \leqslant \overline{v}(x,t) \quad \text{in} \quad \overline{Q}_T.$$

The proof is complete. □

Now we prove finite time blow–up result. We shall suppose that

$$k(x,y,t) \geqslant k_0 > 0, \quad x \in \partial\Omega, \quad y \in \Omega, \quad 0 < t < \beta \tag{2.20}$$

for some positive k_0 and β.

Theorem 2.3. *Let $l > \max\{1, (m+1)/2\}$ and (2.20) hold. Then the solutions of problem (1.1)–(1.3) blow up in finite time for large enough initial data.*

Доказательство. We observe that each solution of (1.1) – (1.3) is a supersolution of the same problem with $a = 0$. Then by Theorem 2.2 and Theorem 2.6 of [18] the solutions of the problem (1.1)–(1.3) blow up in finite time for large enough initial data. □

Remark 2.1. *In the case $q \leqslant m$, $l = (m+1)/2 > 1$ the global existence and blow–up results depend on b and $k(x,y,t)$.*

Remark 2.2. *We note that the proof of Theorem 1.2 in [21] is valid also for $q > 1 > m$. So, a solution of the problem (1.1)–(1.3) blows up in finite time for each $u_0(x) \not\equiv 0$ if $q > \max(m,1)$.*

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